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Electricity Markets & Regulation

Paper 10599_2022

THE UPLIFT PAYMENT ELIMINATION THROUGH THE LAGRANGIAN RELAXATION OF THE REDUNDANT CONSTRAINTS

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Motivation

- The centralized dispatch optimization problem (primal problem) is generally non-convex. Hence, an equilibrium price for power may not exist and uplift payments are needed to compensate power plants for the lost profits.
- Integration of RES to a power market often implies an increased volatility and a need for additional reserves, which contributes to the lost profit of a conventional generation
- A redundant constraint is a constraint that holds on the feasible set of a problem. Previous studies showed that introduction of a linear redundant constraint does not affect maximum profits of the producers but could lower the total uplift payment.
- We investigate if it is possible to construct a set of possibly non-linear redundant constraints that for a given market pricing scheme for power doesn't change the maximum profit of any market player but fully eliminates the need for the uplift payments

Method/Approach

- Introduction of redundant constraints doesn't affect the centralized dispatch outcome (i.e., a set of optimal points of the primal problem)
- The Lagrangian relaxation of a redundant constraint introduces a market price for a new product/service implied by the constraint and the corresponding non-negative revenue for a market player
- The choices of the redundant constraints and the corresponding prices are restricted by a condition that the maximum profit of any market player in unchanged

Objects of investigation

- We consider a multi-period uninode power market with fixed load

Experimental setup & test results

- Linear redundant constraints do not change the market players' maximum profits. A non-linear redundant constraint may fail to have this property.
- The linearity requirement is too restrictive: it may not be possible to attain zero total uplift paument using linear redundant constraints only
- We found that a special type of (possibly non-linear) redundant constraints also has the desired property: any constraint that holds on a set defined by the constraint set of the primal problem with the power balance constraint excluded leaves the market players' maximum profits unchanged. Such constraints may be utilized to eliminate the need for the uplift payments.

Discussion

- Just one (possibly non-linear) redundant constraint is sufficient to produce zero total uplift payment
- The proposal can be utilized for markets with non-linear pricing mechanism
- The results are also applicable to multi-node power systems (provided that FTR-holders are included in a set of market players) with price-sensitive load

Conclusion

- For any given market pricing scheme for power, we explicitly construct a family of the redundant constraints that
    - are additively separable
    - make no effect on the maximum profit of any market player
    - result in zero uplift payment for each market player

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Nomenclature

- *I* – a set of producers
- **d** – demand vector
- $\mathbf{x}_i=(\mathbf{u}_i,\mathbf{g}_i)$, $\mathbf{u}_i$ and $\mathbf{g}_i$ are status and output vectors of unit i
- X_i – private feasible set of unit i
- $C_i(\mathbf{x}_i)$ – cost function of unit i
- **p** – market price (a set of pricing parameters in the case of a non-linear pricing)
- * denotes a primal problem solution
- $^+$ denotes the maximum value of a function

Centralized dispatch optimization problem (primal problem)

$$f^*=\min_{\substack{\mathbf{x}_i\in X_i,\forall i\in I\\ s.t.\sum_{i\in I}\mathbf{g}_i=\mathbf{d}}}\sum_{i\in I}C_i(\mathbf{x}_i)$$

Additively separable redundant constraints on $\times_{i\in I}X_i$

$$h^k(\mathbf{p},\mathbf{x})\le 0\ \ k\in K\quad \forall\mathbf{x}\in\times_{i\in I}X_i$$

$$h^k(\mathbf{p},\mathbf{x})=\sum_{i\in I}h_i^k(\mathbf{p},\mathbf{x}_i)$$

It suffices to consider just one redundant constraint on $\times_{i\in I}X_i$

- the term $\sigma^T h_i(\mathbf{p},\mathbf{x}_i)$

introduced during the Lagrangian relaxation procedure can be viewed as originating from just one constraint

$$\sigma^T h_i(\mathbf{p},\mathbf{x}_i)\le 0$$

- all the redundant constraints can be replaced by just one redundant constraint of the form

$$\sum_{i\in I}N_i(\mathbf{p},\mathbf{x}_i)\ge 0\quad \forall\mathbf{x}\in\times_{i\in I}X_i$$

The total uplift in the presence of the redundant constraint

- a producer i "standard" profit function has the form

$$\pi_i^{st}(\mathbf{p},\mathbf{x}_i)=R_i^{st}(\mathbf{p},\mathbf{x}_i)-C_i(\mathbf{x}_i)$$

- a producer i profit function in the presence of the redundant constraint reads

$$\pi_i(\mathbf{p},v,\mathbf{x}_i)=\pi_i^{st}(\mathbf{p},\mathbf{x}_i)+vN_i(\mathbf{p},\mathbf{x}_i)$$

- the total uplift is expressed as

$$\sum_{i\in I}[\pi_i^+(\mathbf{p},v)-\pi_i^*(\mathbf{p},v)]=f^*-\min_{\mathbf{x}_i\in X_i,\forall i\in I}L(\mathbf{p},v,\mathbf{x})-v\sum_{i\in I}N_i(\mathbf{p},\mathbf{x}_i^*)$$

- to reduce the total uplift we need $\sum_{i\in I}N_i(\mathbf{p},\mathbf{x}_i^*)>0$

Minimizing the total uplift payment

- we require that maximum (i.e., post-uplift) profit of any producer is unchanged

$$\min_{\substack{v\ge 0\\ s.t.\\ \pi_i^+(\mathbf{p},v)=\pi_i^{st,+}(\mathbf{p}),\forall i\in I}}\sum_{i\in I}[\pi_i^+(\mathbf{p},v)-\pi_i^*(\mathbf{p},v)]$$

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continued

General form expression of a redundant constraint that yields zero total uplift

$\sum_{i\in I} N_i(\mathbf{P},\mathbf{X}_i)\geq 0$ with $N_i(\mathbf{P},\mathbf{X}_i)=\min[\pi_i^{st+}(\mathbf{P})-\pi_i^{st}(\mathbf{P},\mathbf{X}_i);\delta_{\mathbf{X}_i,\mathbf{X}_i^*}[\pi_i^{st+}(\mathbf{P})-\pi_i^{st*}(\mathbf{P})]+\gamma_i(\mathbf{P},\mathbf{X}_i)]$

- with an arbitrary non-negative function $\gamma_i(\mathbf{P},\mathbf{X}_i)\geq 0$, $\forall \mathbf{X}_i \in X_i$
- such a choice results in $\nu = 1$

Application to a power market with marginal pricing and fixed unit commitment

- one of the possible choices is given by

$$\sum_{i\in I} N_i(\mathbf{P},\mathbf{X}_i)=\sum_{i\in I}\delta_{\mathbf{X}_i,\mathbf{X}_i^*}[\pi_i^{st+}(\mathbf{P})-\pi_i^{st*}(\mathbf{P})]\geq 0$$

- the corresponding profit function of a producer i has the form

$$\pi_i(\mathbf{P},\mathbf{X}_i)=\mathbf{P}^T\mathbf{G}_i-C_i(\mathbf{X}_i)+\delta_{\mathbf{X}_i,\mathbf{X}_i^*}[\pi_i^{st+}(\mathbf{P})-\pi_i^{st*}(\mathbf{P})]$$

- for a single period power market such a redundant constraint (up to a constant factor) equals
  - $u_i \leq 1$ for an offline unit i
  - $u_i \geq 0$ for an online unit i

Convex-hull pricing (CHP) method

- the CHP uplift term can be expressed as originating from the redundant constraint

$$\sum_{i\in I} N_i(\mathbf{P}^{CHP},\mathbf{X}_i)\geq 0$$

with

$$N_i(\mathbf{P}^{CHP},\mathbf{X}_i)=\delta_{\mathbf{X}_i,\mathbf{X}_i^*}[\pi_i^{st+}(\mathbf{P}^{CHP})-\pi_i^{st*}(\mathbf{P}^{CHP})]$$

Conclusion

- The Lagrangian relaxation of the redundant constraints that holds on $\forall \mathbf{X} \in \times_{i\in I} X_i$

does not change the maximum values of the producer profits and may reduce the total uplift

- These constraints are redundant not only on the feasible set of the primal problem but also on a larger set $\times_{i\in I} X_i$
- To attain zero total uplift payment, It suffices to introduce just one (additively separable) redundant constraint
- We obtain the general form expression of such a (possibly non-linear) redundant constraint parameterized by an arbitrary non-negative function
- All the redundant constraints that have the required properties can be obtained from the general form expression
- The market pricing for power is considered as given, so that an introduction of the redundant constraint has no effect on the pricing for power
- In the case of CHP method, an introduction of the redundant constraint results in the same set of market prices but fully absorbs the uplift payment terms resulting in zero total uplift
- the proposed approach is also applicable to markets with price-sensitive consumers and any pricing scheme that expresses revenue (charge) for power as a function of price parameters and the state-output (consumption) variables of a producer (consumer)