

**B1 – Insulated Cables****PS 1 – Learning from experiences – Design, manufacturing, installation techniques, maintenance and operation****Determination of soil thermal resistance: A holistic approach**

**Andreas I. CHRYSOCHOS\***  
Hellenic Cables  
Greece  
anchryso@gmail.com

**Dimitrios CHATZIPETROS**  
Hellenic Cables  
Greece  
dchatzipetros@hellenic-cables.com

**Varvara RIZOU**  
Hellenic Cables  
Greece  
vrizou@hellenic-cables.com

**Konstantinos PAVLOU**  
Hellenic Cables  
Greece  
kpavlou@hellenic-cables.com

**Konstantinos TASTAVRIDIS**  
Hellenic Cables  
Greece  
ktastavridis@hellenic-cables.com

**George GEORGALLIS**  
Hellenic Cables  
Greece  
ggeorgal@hellenic-cables.com

**SUMMARY**

A new, holistic, accurate and computationally efficient methodology, which employs the finite element method, is presented in this paper for the calculation of the cable external thermal resistance. Previous works in literature deal with similar topics by employing the superposition principle. Although the latter may be securely assumed when cables are laid spaced, it becomes rather questionable when cables get closer to one another, thus limiting the applicability of the relevant methods. The methodology proposed in this paper covers not only spaced cables, but also cables being laid in close physical proximity, where the superposition principle becomes questionable. The thermal resistance is evaluated based on the temperature rise of the cable surface with respect to ambient conditions and the amount of heat dissipated to the surrounding soil. The so-called ‘thermal conductance matrix’  $G$  is developed in a column-wise manner, from which both self and mutual thermal resistance terms are then extracted. The proposed methodology is universal and applicable to any material and shape of multilayer or backfilled soil, taking also into account the thermal impact of adjacent cables, irrespective of how close they are placed. Hence, overcoming a limitation of the existing methods, which can only be used for non-touching cables, the proposed one is applicable to both touching and non-touching cables. Its computational burden is small, since the heat conduction problem is solved only in the surrounding soil and not inside the cable. The method is fully compatible with the steady state current rating methodology of the IEC 60287 Standard, thus it may be further used for the calculation of the cable ampacity for both land and subsea applications.

**KEYWORDS**

Backfill - Cable - Current Rating - Finite Element Method - Non-Uniform Soil - Thermal Resistance

## 1. INTRODUCTION

The demand for submarine and underground cables has substantially increased in recent years due to environmental, economic and aesthetics reasons. The high manufacturing, installation and operating costs of cables have led to the systematic optimization of their design with primary goal the increase of current carrying capacity. Several factors, such as the soil thermal resistance, can influence and severely limit the amount of the heat dissipated from the cable, resulting in either reduced current carrying capacity for a given design or increased total cable cost due to larger conductor sizes required.

The importance of accurate determination of the soil thermal resistance is even more pronounced in the current rating calculations of modern cable projects. In such cases, the cable surroundings often include multilayer soil configurations with different thermal properties or soil sections enhanced with backfilling materials of lower thermal resistivity in order to improve the heat dissipation from the cable. In addition, the thermal impact of other adjacent cables installed in these multilayer soil structures must also be taken into account as accurately as possible.

With respect to steady-state thermal conditions, several analytical and numerical approaches have been proposed to determine the thermal resistance of the surrounding soil. The former include closed mathematical expressions that are based on the multiple reflections of heat sources and their images [1], [2]. The numerical methods typically rely on the technique of conformal transformation with the use of finite differences [3], [4], and are applicable to any geometry of multilayer or backfilled soil. Recently, some of the authors have presented a new numerical method based on the superposition principle by employing the finite element method (FEM) [5]. This method effectively tackles several of the drawbacks of the previous approaches while being still compatible with the mathematical formulation of the IEC Standard 60287-2-1 [6].

However, despite their wide range of applicability, the abovementioned approaches also exhibit certain limitations. The application of the superposition principle for the calculation of the mutual elements in the thermal resistance matrix results in reduction of accuracy, especially in cases where the cables are placed in close physical proximity. In those cases, the thermal fields induced by the several adjacent cables affect one another, making the principle of superposition rather questionable. As a result, the applicability of those methods is actually limited to non-touching cables, i.e., where the clearance is typically of the order of one cable diameter or more.

This paper presents a new, holistic approach to the determination of the soil thermal resistance, which overcomes the aforementioned limitations of the superposition approach. A heat transfer problem under steady-state conditions is set in FEM, while defining arbitrary temperatures to all cables and the soil surface. The calculation is performed by determining the so-called ‘thermal conductance matrix’ without resorting to the superposition principle. Its elements are calculated column-wise by computing all ratios between the amounts of heat dissipated by all cable surfaces and the temperature rise of one cable in turn, assuming zero temperature rise of all others. By inverting the ‘thermal conductance matrix’, the matrix of all thermal resistances is obtained, which can be further used for the calculation of the cable current capacity using the IEC Std. 60287-2-1 [6].

The paper structure is as follows: A short literature review is first made, briefly presenting the existing methods; the proposed method is subsequently presented; the latter is then compared with the existing methods, including the approximating formulae of the IEC Std. 60287-2-1 [6] and the approach of [5]; finally, numerical results for touching and non-touching cables, in both external resistance and current rating terms are shown.

## 2. CALCULATION OF EXTERNAL THERMAL RESISTANCE

### 2.1 Existing methods

Based on Electra No. 98 [4], the temperature rise of the surface of cable  $i$  under steady-state thermal conditions is given by:

$$\Delta\theta_i = \theta_i - \theta_0 = \sum_{j=1}^n R_{ij}Q_j \quad (1)$$

where  $\theta_i$  the temperature of cable surface  $i$  [ $^{\circ}\text{C}$ ],  $\theta_0$  the temperature of soil surface known as ambient temperature [ $^{\circ}\text{C}$ ],  $Q_j$  the total power losses per unit length of cable  $j$  [ $\text{Wm}^{-1}$ ],  $R_{ii}$  the self thermal resistance of cable  $i$  [ $\text{KmW}^{-1}$ ], and  $R_{ij}$  the mutual thermal resistance between cables  $i$  and  $j$  [ $\text{KmW}^{-1}$ ]. The abovementioned formulation is valid when the surface of the cable of interest and the soil surface are assumed isothermal, with the latter condition being also known as the Kennelly hypothesis [7].

The thermal resistance matrix  $R$  of (1) resembles the external thermal resistance  $T_4$  of one cable in the IEC Std. 60287-2-1 (§4.2), but also covers the generic case of multiple unequally loaded cables. More specifically, the diagonal element  $R_{ii}$  is comparable to the quantity  $T_4$  of a single isolated cable in the IEC Std. 60287-2-1 (§4.2.2) and relates to the temperature difference between the external surface of that isolated cable and the soil surface. The off-diagonal element  $R_{ij}$  ( $i \neq j$ ) is comparable to the term  $\Delta\theta_{kp}/W_k$  in the IEC Std. 60287-2-1 (§4.2.3.2) and relates to the temperature difference between the center of the cable  $i$  and the soil surface due to the heat dissipated by the cable  $k$ .

Although the existing methods account effectively for the thermal resistance the surroundings impose on the cable, such as the IEC Std. 60287-2-1 [6] for uniform soils or the Electra method [4] and the approach of [5] for non-uniform soils, they still rely, in their generic case, on the superposition principle and the limitations this principle introduces. The method presented in the next section demonstrates an approach which overcomes these limitations.

## 2.2 Proposed method

Various methodologies can be found in the literature which calculate directly the thermal resistance matrix  $R$  of (1) by exploiting the superposition principle [1]-[5]. Due to this assumption, the calculation of  $R$  becomes straightforward even in multilayer or backfilled soil [5], however, it suffers from limited accuracy especially in cases where the cables are placed in close physical proximity. In those cases, the thermal fields induced by the several adjacent cables alter one another, making the principle of superposition rather questionable [4]. As a result, the applicability of those methods is actually limited to non-touching cables.

To overcome the aforementioned limitations, a new, holistic approach is proposed which determines the so-called ‘thermal conductance matrix’  $G$  [ $\text{WK}^{-1}\text{m}^{-1}$ ] by employing FEM and without resorting to the superposition principle [4]. Within this context, it holds:

$$Q = G\Delta\theta \quad (2)$$

where  $\Delta\theta$  [ $^{\circ}\text{C}$ ] the column vector of temperature rise of all cable surfaces with respect to temperature soil surface, and  $Q$  [ $\text{Wm}^{-1}$ ] the column vector of total power losses per unit length of all cables.

A typical configuration of 2 adjacent cables, randomly laid, is shown in Figure 1. Cables  $i$  and  $j$  with external diameters  $D_{e,i}$  [m] and  $D_{e,j}$  [m] are buried at depths  $h_i$  [m] and  $h_j$  [m], respectively, within an arbitrary multilayer soil of varying thermal resistivity. The problem of heat transfer in solids under steady-state conditions is given by Fourier’s conduction law and solved with FEM. The boundary condition at the top soil surface (blue line) is set to a Dirichlet isothermal of arbitrary temperature  $\theta_0$ , based on the Kennelly hypothesis [7]. Similarly, for the calculation of column  $i$  of  $G$ , the cable surface (red solid circle) of the corresponding cable is set to an arbitrary temperature  $\theta_i$  ( $\theta_i \neq \theta_0$ ) while all the other cable surfaces are set to the ambient temperature  $\theta_0$  ( $\theta_j \equiv \theta_0$ ). The imposition of boundary

conditions to the rest of the edges (green lines) is somewhat more challenging, since the temperature field can be considered unbounded. In order to limit the extent of the FEM model to a manageable region of interest with reasonable execution time, a coordinate scaling is adopted to layers of virtual domains surrounding the physical region of interest [8], i.e., to the upward diagonal regions shown in Figure 1. These virtual layers can be mathematically stretched out towards infinity, where the net heat flux  $\mathbf{q}$  is imposed equal to zero as a Neumann boundary condition with  $\mathbf{n}$  being the normal unit vector. Consequently, the model becomes computationally efficient while the solution inside the region of interest is not affected by the artificial geometric boundaries.

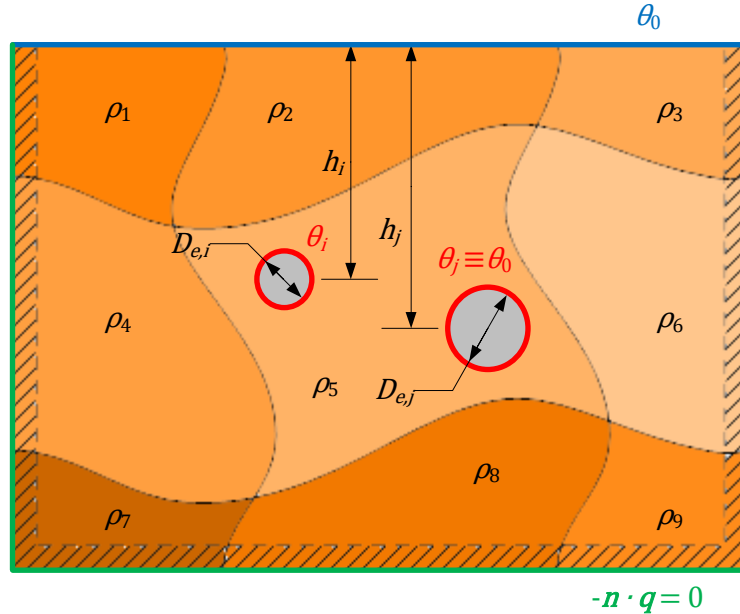


Figure 1 : Indicative FEM representation of multilayer soil with 2 randomly laid adjacent cables. Inclusion of appropriate boundary conditions for the calculation of column  $i$  in matrix  $G$ .

After the solution of the thermal problem via the Galerkin method [9], the per unit length heat loss  $Q$  [ $\text{Wm}^{-1}$ ] dissipated or absorbed by each cable is calculated by numerical integration. Then, the column  $i$  of the thermal conductance matrix  $G$  is determined by:

$$G_{ii} = \frac{Q_i}{\Delta\theta_i} = \frac{Q_i}{\theta_i - \theta_0} \quad (3)$$

$$G_{ji} = -\frac{Q_j}{\Delta\theta_i} = -\frac{Q_j}{\theta_i - \theta_0} \quad (4)$$

The elements of matrix  $G$  are computed in a column-wise manner by calculating all ratios between heat losses in all cables and the temperature rise in one cable in turn, while the temperature rises of the other cables being assumed equal to zero. Due to the cable and installation symmetry, matrix  $G$  may have some sort of symmetry, thus less elements need to be calculated, reducing further the total computational cost. As a final step, by inverting matrix  $G$ , the thermal resistance matrix  $R$  of (1) is obtained. Calculating heat loss  $Q$  in a way that the disturbance of the involved thermal fields is accounted for makes the proposed method capable of considering accurately enough cables being placed in close physical proximity. The proposed method is tested against the existing methods in the next section.

### 3. NUMERICAL RESULTS

The performance of the proposed method is demonstrated in this section for different installation cases of buried underground cables. Results are compared with those obtained by both analytical [6] and

numerical approaches [5] found in the literature. The accuracy of the proposed method is assessed in terms of the relative difference  $err$  of (5), which is calculated on the total value of the thermal resistance for each cable core.

$$err = \frac{R^{proposed} - R^{other}}{R^{proposed}} 100\% \quad (5)$$

A typical underground cable is considered with external diameter  $D_e$  equal to 100 mm [5]. The cable is installed in the three different scenarios shown in Figure 2 assuming a fixed burial depth  $h$  [m] and a varying axial separation  $s$  [m]. Without loss of generality, the soil is assumed homogeneous in this section.

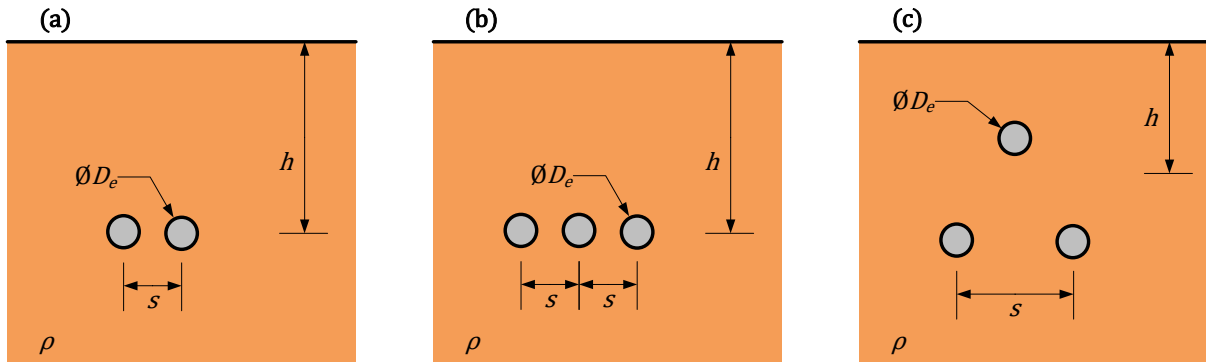


Figure 2: (a) Two single-core cables in flat formation, three single-core cables in (b) flat and (c) trefoil formation.

### 3.1 Two single-core cables in flat formation

Since the two cable cores are thermally coupled, the matrices  $G$  and  $R$  are both of order  $2 \times 2$ . In addition, due to the cable symmetry, only one column of  $G$  needs to be calculated, as shown in (6).

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{12} & G_{11} \end{bmatrix} \rightarrow R = \begin{bmatrix} R_{11} & R_{12} \\ R_{12} & R_{11} \end{bmatrix} \quad (6)$$

Figure 3a-b present  $R_{11}$  (self) and  $R_{12}$  (mutual) elements of  $R$  with respect to the axial separation  $s$ . Results by the proposed method are compared with the corresponding derived by the approach of [5] and the IEC Std. 60287-2-1 (§4.2.3.3.1). It is apparent that the results derived by the latter two methods coincide. As shown in [5], this is expected in homogeneous soils for non-touching cables, since both methods rely on the superposition principle for the calculation of the thermal resistance matrix  $R$ . The self term is independent of axial separation, thus remains constant, while the mutual term decreases with increasing  $s$ . On the other hand, both terms derived by the proposed method deviate as the axial separation decreases. This happens since the proposed method solves the problem in a holistic way, without resorting to the superposition principle, thus the self term also depends on axial separation  $s$ , since it is influenced by the adjacent cable. For larger axial distances, where the thermal field induced by the adjacent cable is not profound, all methods converge.

Figure 3c shows the sum of the row elements, which is also compared with the IEC Std. 60287-2-1 (§4.2.4.1.1) for the specific case of two touching single-core cable in flat formation. It is evident that for touching cables, i.e., for  $D_e$  equal to 0.1 m, the proposed method yields the same result as the IEC Std. 60287-2-1 (§4.2.4.1.1), highlighting its high accuracy even in cases of touching cables. Finally, in Figure 3d, the relative error of (5) between the proposed method and the approach of [5] is calculated. It can be concluded that the approach of [5] yields more conservative values compared to the proposed method for nearly touching cables. For larger separation distances, the error tends to zero and the two methods converge.

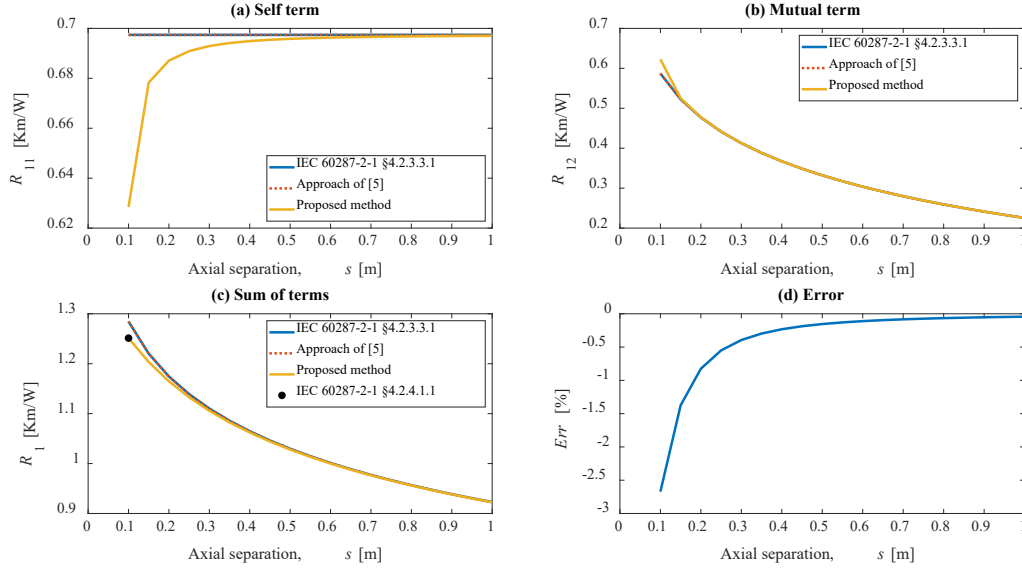


Figure 3: Two single-core cables in flat formation. (a) Self term  $R_{11}$ , (b) mutual term  $R_{12}$ , (c) sum of the thermal resistance matrix, and (d) relative error with respect to axial separation.

### 3.2 Three single-core cables in flat formation

In the case of three cable cores, the matrices  $G$  and  $R$  are both of order  $3 \times 3$ . Due to the symmetry imposed by the flat formation, only two columns of  $G$  need to be calculated, as shown in (7).

$$G = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{12} & G_{22} & G_{12} \\ G_{13} & G_{12} & G_{11} \end{bmatrix} \rightarrow R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{12} & R_{22} & R_{12} \\ R_{13} & R_{12} & R_{11} \end{bmatrix} \quad (7)$$

Figure 4 presents the elements  $R_{11}$ ,  $R_{22}$ ,  $R_{12}$ , and  $R_{13}$  with respect to the axial separation  $s$ , respectively. Results present the same behavior as in the case of two single-core cables in flat formation. Specifically, the approach of [5] coincides with the IEC Std. 60287-2-1 (§4.2.3.3.1), while the proposed method deviates for relatively small axial distances, where the thermal field induced by the adjacent cable cores influences the examined one.

Figure 5a-b show the sum of the row elements for both cable cores, which are again compared with the IEC Std. 60287-2-1 (§4.2.4.1.1) for the specific case of three touching single-core cable in flat formation. Results are in good agreement, validating the proposed approach. Compared to the approach of [5], the proposed method yields less conservative results in terms of external resistance  $T_4$  for all cases, which can be also confirmed by the negative relative error of (5) in Figure 5c.

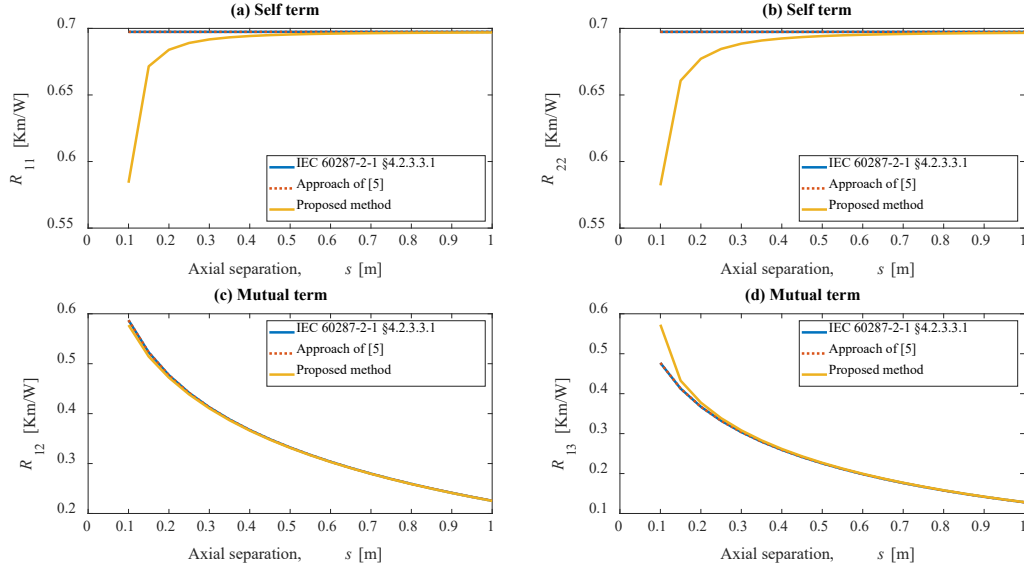


Figure 4: Three single-core cables in flat formation. Element (a)  $R_{11}$ , (b)  $R_{22}$ , (c)  $R_{12}$ , and (d)  $R_{13}$  of the thermal resistance matrix with respect to axial separation.

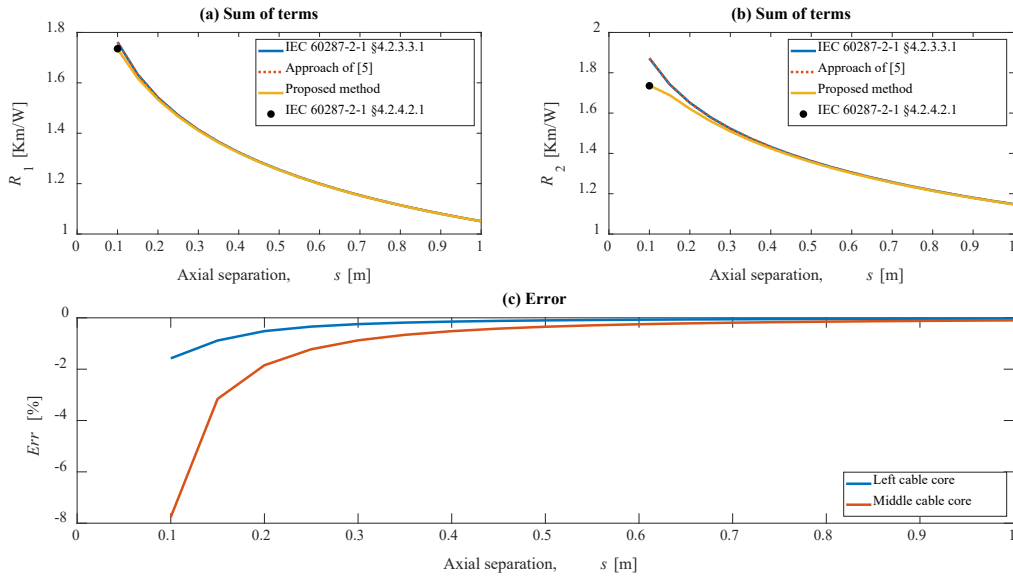


Figure 5: Three single-core cables in flat formation. Total thermal resistance of (a) left and (b) middle cable with respect to axial separation. (c) Relative error for both cable cores.

### 3.3 Three single-core cables in trefoil formation

In the case of three cables cores in trefoil formation, the structure of matrices  $G$  and  $R$  is the same as in the case of flat formation.

$$G = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{12} & G_{22} & G_{12} \\ G_{13} & G_{12} & G_{11} \end{bmatrix} \rightarrow R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{12} & R_{22} & R_{12} \\ R_{13} & R_{12} & R_{11} \end{bmatrix} \quad (8)$$

Similar trend for the elements of matrix  $R$  is observed in Figure 6. The non-monotonic behaviour of element  $R_{22}$  calculated by the proposed method is due to the competitive thermal impact between the adjacent cable cores and its burial depth for increasing axial separation. Figure 7a-b present the total

thermal resistance of the bottom-left and upper-middle cable cores, while Figure 7c shows the relative error between the proposed method and the approach of [5].

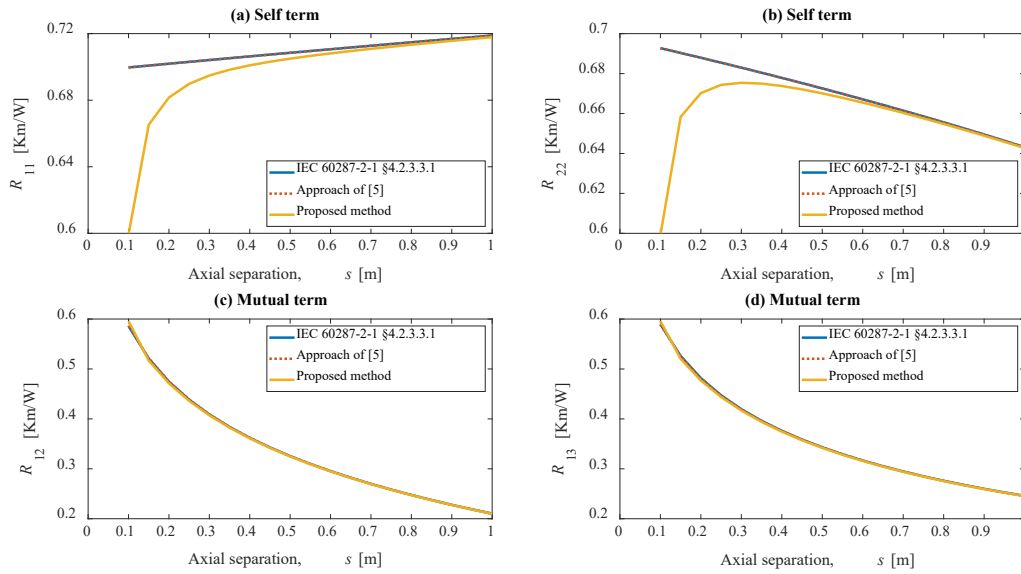


Figure 6: Three single-core cables in trefoil formation. Element (a)  $R_{11}$ , (b)  $R_{22}$ , (c)  $R_{12}$ , and (d)  $R_{13}$  of the thermal resistance matrix with respect to axial separation.

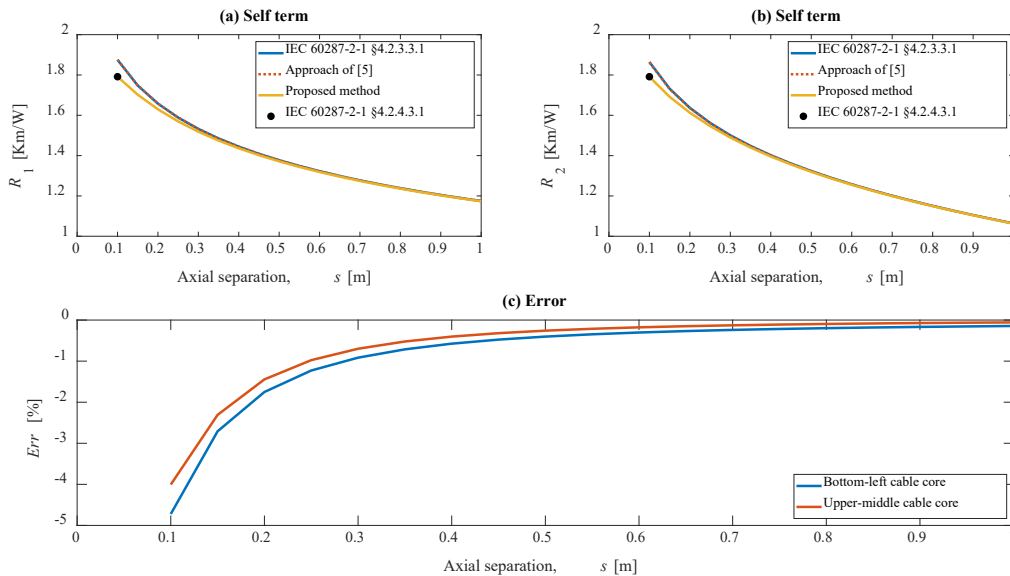


Figure 7: Three single-core cables in trefoil formation. Total thermal resistance of (a) bottom-left and (b) upper-middle cable with respect to axial separation.

#### 4. EFFECT ON CURRENT RATING

The calculation of the external thermal resistance  $T_4$ , based on the method proposed in this paper, is presented and assessed in previous sections. Although this assessment relies on the comparison between various methods, the effect of  $T_4$  on current rating gives a more meaningful and informative outcome with reference to the cable performance.

The cable current rating assuming a maximum design temperature of  $90^\circ\text{C}$  for a  $1200\text{ mm}^2$  cable with aluminum conductor and  $D_e$  equal to  $0.1\text{ m}$  is calculated in this section for the various methods already discussed. To evaluate the applicability and demonstrate the superiority of the proposed method in the



generic case of non-uniform soils, a flat formation of cables enclosed in a backfill such as that of Figure 8 is considered. Since axial separation  $s$  varies, parameter  $c$  [5] also varies. The middle cable is expected to be the hottest one; hence, current rating only for that cable is performed. Cross-bonding configuration with equal minor sections is assumed for metallic sheaths and the relevant cable losses are imported in accordance to the IEC Std. 60287-1-1.

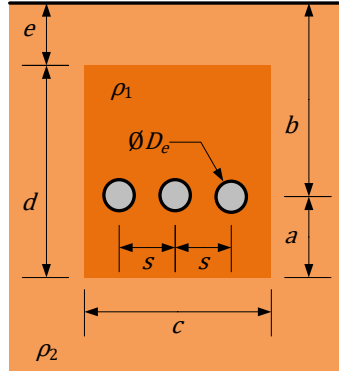


Figure 8: Three single-core cables in flat formation installed within a backfill. Parameters are set to  $\rho_1 = 1 \text{ KmW}^{-1}$ ,  $\rho_2 = 3 \text{ KmW}^{-1}$ ,  $a = 150 \text{ mm}$ ,  $b = 2000 \text{ mm}$ ,  $d = 500 \text{ mm}$ ,  $e = 1100 \text{ mm}$ .

Current rating results for the several methods discussed in the present paper are shown in Figure 9a for various  $s$  values. The relative error in terms of current is also presented in Figure 9b based on (5). Current rating values between the proposed method and the approach of [5] appear to deviate when the cables are considered in closer physical proximity, as expected also from Figure 5b. Instead, they almost coincide with each other for larger spacing values, thus verifying the applicability of the proposed method for any axial separation  $s$ . In addition, the relative error between the proposed and the IEC Std. 60287-2-1 in Figure 9 is remarkable, since the latter cannot take into account of the actual, multilayer soil thermal resistance. Thus, the worst thermal resistivity between native soil and backfill must be considered for  $T_4$  calculation, i.e., that of  $\rho_2$  equal to  $3 \text{ KmW}^{-1}$ . For this reason, significantly more conservative ampacity values occur when using the IEC Std. 60287-2-1 [6] method compared to the method proposed in the paper, which proves to be superior even when non-uniform soils are considered. It is noticeable that a conductor size of  $1800 \text{ mm}^2$  instead of  $1200 \text{ mm}^2$  would be required to reach the same ampacity value for the maximum error shown in Figure 9b, i.e., a conductor larger up to three standardized sizes.

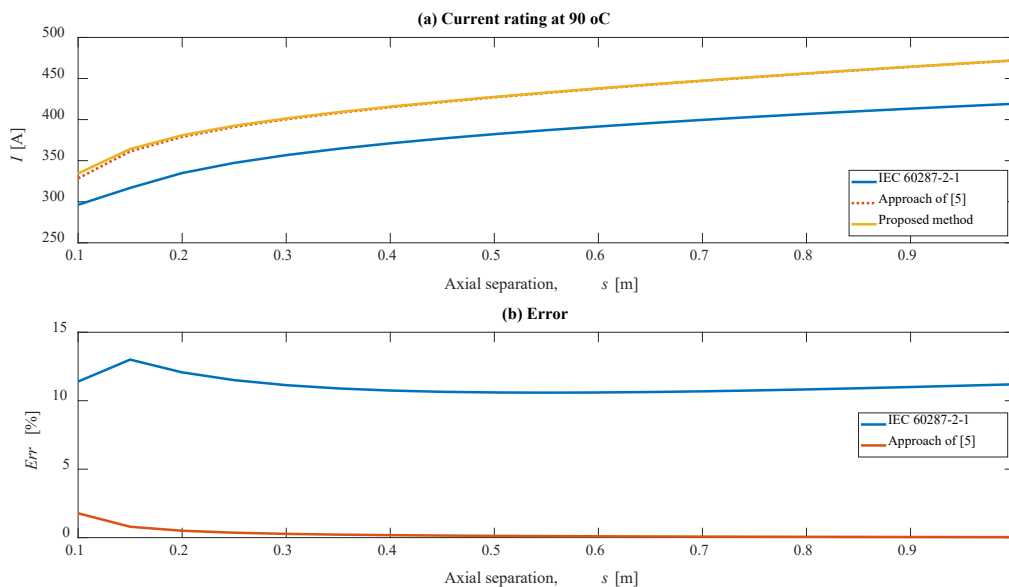


Figure 9: Three single-core cables in flat formation within a backfill as per Figure 5 in [5]. (a) Current rating and (b) relative error between existing and proposed methods.

## 5. CONCLUSIONS AND FUTURE WORK

A holistic approach for calculating the external cable thermal resistance,  $T_4$ , is presented in this paper. Based on the so-called ‘thermal conductance matrix’,  $G$ , and the use of FEM, the proposed method is demonstrated to tackle certain limitations imposed by previous methods, which rely on the superposition principle, and proves to be universal, without any reduction in accuracy. Although the previous works, such as that of [5], may be effectively used for non-touching cables, results indicate that lower  $T_4$  values occur for touching cables compared to the approach of [5]. By comparing  $T_4$  for various axial separation  $s$  values, the wide applicability of the proposed method is confirmed, i.e., including touching and non-touching cable installations. The superiority of the proposed method is also proven in current rating terms, particularly compared to the conventional IEC Std. 60287-2-1 method which is incapable of considering multilayer soils. Significantly higher ampacity occur in this case, thus demonstrating the increased design optimization margins offered.

Despite its wide range of applicability, the proposed method also includes certain inherent limitations. For instance, it does not deal with nonlinear phenomena, such as the effect of moisture migration and the location of boundary between moist and dry material. In addition, the proposed method is restricted to steady-state thermal conditions, by assuming that transient changes due to thermal capacitances of cables and soil have been completed. A solution to this latter point will be presented in future papers.

## BIBLIOGRAPHY

- [1] V. T. Morgan P. Slaninka “The external thermal resistance of power cables in a group buried in non-uniform soil” (Electric Power Systems Research, volume 29, 1994, pages 35-42)
- [2] P. Slaninka V. T. Morgan “External thermal resistance of power cable in nonuniform soil” (IEE Proceedings-A, volume 139, number 3, 1992, pages 117-124)
- [3] G. Luoni A. Morello H. W. Holdup “Calculation of the external thermal resistance of buried cables through conformal transformation” (IEE Proceedings, volume 116, 1969, pages 1885-1890)
- [4] Working Group SC 23-02 CIGRE. “The calculation of the effective external resistance of cables laid in materials having different thermal resistivities” (Electra, number 98, 1985, pages 19-42)
- [5] A. I. Chrysochos et al “Rigorous calculation of external thermal resistance in non-uniform soils” (CIGRE, France, 2020)
- [6] Standard 60287-2-1 IEC. “Electric cables - Calculation of the current rating - Part 2-1: Thermal resistance - Calculation of the thermal resistance” (IEC, 2015, pages 1-84)
- [7] A. E. Kennelly “On the carrying capacity of electric cables submerged, buried or suspended in air” (Annual Meeting of Association of Edison Illuminating Companies, USA, 1893)
- [8] A. I. Chrysochos et al “Capacitive and inductive coupling in cable systems - Comparative study between calculation methods” (JICABLE, France, 2019)
- [9] M. M. Vajnberg “Variational methods and methods of monotone operators in the theory of nonlinear equations” (Wiley, USA, 1974)