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Probabilistic safety concept in OHL construction

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SUMMARY

The decision to push ahead with the energy transition in Germany and the associated shutdown of nuclear and fossil fuel power plants poses major challenges, especially for the transmission grid. This must be expanded for the future transport of energy. This involves, on the one hand, the construction of DC links to connect offshore wind farms and, on the other hand, the expansion and refurbishment of the existing AC grid. In the context of the rehabilitation of the existing network, the application of a probabilistic safety concept offers an interesting technical approach to the implementation of these challenges. Both the current [1] and predecessor VDE 0210 (regulations for the construction of overhead power lines in Germany) do not take into account the level of possible damage consequences for high-voltage steel lattice pylons in Germany since 1903. Thus, the same probability of failure is specified for all pylon locations, both in densely populated and undeveloped areas.

This paper describes the main features of the probabilistic calculation method for existing high-voltage steel lattice pylons based on DIN EN 1990 [2], as it is principally regulated for overhead lines in VDE-AR-N 4210-4 [3]. By evaluating the possible consequences of damage, all pylon locations can be assigned to 5 different reliability levels. With the help of stochastic models, impacts and resistances can be described very accurately. In this way, a site-specific proof of stability can be provided for existing steel lattice pylons. This enables a risk-considered refurbishment of the energy transmission grid. As a starting point in this context, reliability level 5 represents all series of standards. Based on this, the probabilistic model significantly increases the safety level of existing pylons.

Within the framework of a case study, it was possible to demonstrate for the load case of pressure failure that the reliability index of 3.6 almost reaches reliability level 2. If, in the context of this example, the pylon was located near a railway line, it would have been assigned reliability level 3. With 3.6, this pylon would be clearly above the requirements and the proof would thus be provided.

The probabilistic safety concept offers great technical potential, as there are further optimisation possibilities through more precise considerations on the side of the impacts (e.g. the weather parameters). The development of the probabilistic safety concept for lattice pylons can be applied to other areas of the energy grid.

KEYWORDS

overhead line construction, probabilistic safety concept, resource-efficient energy grid, optimal safety level

1. Introduction

The energy transition in Europe with the goal of climate neutrality is in full swing. The provision of a sustainable and efficient energy transmission network plays a major role in this. In this context, effort and material should be used as efficiently as possible with the aim of technical system safety.
For high-voltage lattice pylons, since 1903, various editions of VDE 0210 (regulations for the construction of overhead power lines in Germany) have considered the level of possible damage consequences to be identical for all locations, thus aiming for the same probability of failure.
Since 2010, the probabilistic calculation method and normative basis from DIN EN 1990 [2] have made it possible to assign all pylon locations to different reliability levels by evaluating possible damage consequences. By adhering to the individually determined reliability indices according to VDE-AR-N 4210-4 [3], a low-risk repair of the steel lattice pylons is guaranteed.

2. Probabilistic safety concept

2.1. Classification of the safety concepts

The various safety concepts are shown in Table I [4].

Table I: Classification of the safety concepts

Safety concept	Level	Value for the reliability	Design requirement
Deterministic (empiric)	0	Global safety factor ν	existing $\sigma \leq$ allowable σ = critical σ / ν
Semi-probabilistic	I	Partial safety factors γ_R, γ_S	existing σ ($\gamma_S \cdot S$) $\leq$ critical σ / γ_R
Probabilistic (approximation)	II	Reliability index β	existing $\beta \geq$ required β
Probabilistic (exact)	III	Probability of failure P_f	existing $P_f \leq$ allowable P_f

The first safety concept (Level 0) was based on gathering experience. For the design of structures, stresses from frequently undershot actions were assumed in 1-fold height. On the resistance side, rarely undershot stresses were assumed, which were divided by a global safety factor ν. This had to include all uncertainties on the action and resistance side. In this way, it was not possible to do justice to the different scattering effects. In OHL construction, the wind load scatters more strongly than compared to the conductor tension force. The level of the safety factor was initially determined intuitively. The level was influenced by the material used, to which it referred, or the circumstance of whether damage had been announced or was to be expected suddenly. If no damage occurred over a long period of time in structures designed with this safety factor, there was a suspicion of safety reserves and the safety factor could be gradually lowered to avoid wasting resources. If, on the other hand, damage occurred frequently, the factor had to be increased because the risks were too high. Nowadays, the second, common safety concept (Level 1) uses partial safety factors. These refer to quantile values of certain random realisations of the action and resistance side. High realisations of the actions, e.g. the 0.98 quantile, which is fallen short of or at most reached with a probability of 0.98, are increased by γ_S.
Low realisations of the material strength, e.g. the 0.05 quantile, are decreased with γ_R. With this approach, it is theoretically possible to take into account the different degrees of dispersion of the various variables on the action and resistance side and to control the probability of failure by specifying a certain distance between stress due to 0.98 quantile of action and 0.05 quantile of strength. For reasons of simplification, the number of partial safety factors is kept low. Their amount must therefore be oriented towards the most scattering variables. On the other hand, it is difficult for a society to quantify the accepted probability of failure, i.e. the probability with which the stress due to the randomly realised

actions exceeds the randomly realised strength in the cross-section. Finally, the previous design practice should not be suddenly discontinued. Structures designed with global safety are still in operation today and it would be incomprehensible if half or twice as large cross-sections were designed due to a change in the safety concept.
Despite its weaknesses, this safety concept will be used for the foreseeable future in the construction of new structures. For the recalculation of existing structures, on the other hand, the application of probabilistic safety concepts Level II or III is worthwhile. With the Level II concept, a minimum value of the reliability index β must be maintained; with the Level III concept, the calculated probability of failure P_f must not exceed a limit value. The Level II concept is an approximation method, which is presented in more detail below. An exact calculation of the failure probability P_f according to Level III is not possible in all cases, the approximation according to Level II is generally possible. While the cross-section dimensions or the number of connecting elements can be determined relatively unproblematic for the new construction of structures, a partial safety factor that is set too high for simplification reasons may lead to a replacement of the supposedly undersized cross-section for recalculations in the existing structure. The replacement with higher dimensioned components is more expensive than the use of the same component in new constructions. In Figure 1, this can be seen in a line of reinforcement costs that grows faster than in new constructions.

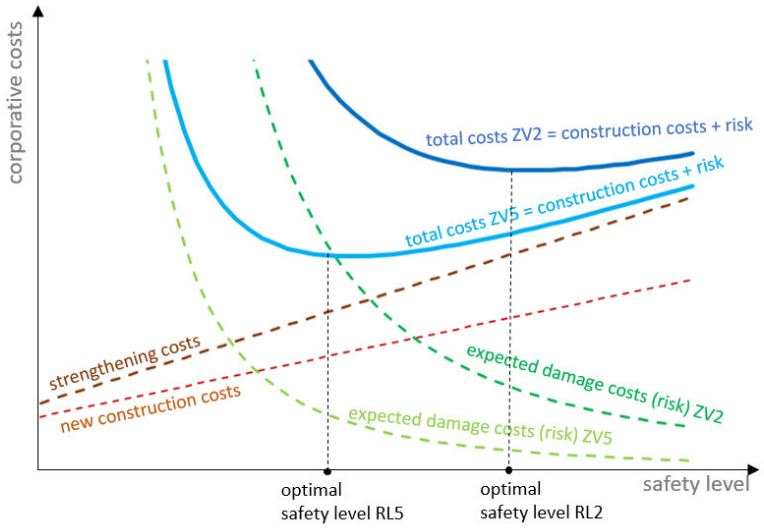

Figure 1: optimal safety level

The reasons are the re-establishment of a construction site, anchoring work and the possible necessity for the decommissioning of the OHL. The construction work can also damage the structure by introducing constraints. By dividing into areas with different damage consequences in the course of the OHLs, it is also possible to use resources preferentially in areas where only very small probabilities of failure can be accepted, instead of using the same resources in all areas regardless of their damage consequences.

2.2. Reliability level for towers

From Figure 1 it can be seen that with increasing consequential damage costs, the minimum of the total costs shifts to high optimal safety levels despite increasing reinforcement costs.
VDE-AR-N 4210-4 distinguishes five safety levels with target reliability indices according to Table II.

Table II: Safety levels and target safety indices

Required reliability indices of the pylon system (according to table 7 VDE-AR-N4210-4:2014-08)				
Reliability level	**required $\beta_{Sys,akk}$**	**permitted $P_{f_{Sys,akk}}$**	**Site-dependent hazard**	**examples**
1	4.3	$8.5 \cdot 10^{-6}$	very high	stadiums
2	3.8	$7.2 \cdot 10^{-5}$	High (reference level)	motorways
3	3.3	$4.8 \cdot 10^{-4}$	Medium	Railway station building
4	3.0	$1.35 \cdot 10^{-3}$	Low	Parking spaces
5	2.6	$4.7 \cdot 10^{-3}$	Very low	Forest area

The target reliability index of β = 3.8 of the reference level was obtained by the reliability-theoretical recalculation of a large number of pylons designed according to EN 50341-2-4 [1] on the basis of 1-year extreme value distributions. This is followed above by reliability level 1, in which the accepted probability of failure had to be reduced by about one order of magnitude due to significantly higher damage consequences compared to the reference level. Below the reference level, reliability levels 3, 4 and 5 follow, in which higher probabilities of failure could be accepted due to smaller consequences of damage.

2.3. Verification procedure

The verification is carried out by comparing the existing reliability index (existing β_{Sys}) of the technical structure, which is directly related to the failure probability $P_{f_{Sys}}$, with the site-dependent required reliability index (required β_{Sys}).

Each component of a steel lattice pylon can lose its function due to several failure mechanisms, which are described by components. According to the pessimistic assumption of a series system, the entire system fails if a single component fails.

A component failure can be caused, for example, by tensile, compressive, shear, hole friction or block failure.

2.4. Limit state functions

The limit state function is the difference function between capacity, i.e. the resistance r($\underline{x}$) of a component or fastener in structural mechanics, and the stress consuming it, i.e. the stress or internal force in the cross-section s($\underline{x}$) in structural mechanics, caused by external actions. Each failure mechanism corresponds to a component of the structure. Resistance and stress, and thus also the limit state function itself, depend on generally several random variables $\underline{x}$. Their magnitude is not precisely known, only that of the limit state function. Their magnitude is not known exactly, only the probabilities with which certain variable realisations are undercut or maximally achieved. If the variables are realised in such a way that g($\underline{x}$) = r($\underline{x}$) - s($\underline{x}$) > 0, survival is present. If the variables are realised in such a way that g($\underline{x}$) = r($\underline{x}$) - s($\underline{x}$) < 0, failure is present. The probability with which < 0 occurs is the interesting failure probability P_f (see Figure 2). For lattice steel pylons, four failure possibilities can be defined in a simplified way in accordance with VDE-AR-N 4210-4: Compression, tension, shearing and pitting. Thus, the following limit state functions result:

$$g_D = r_D(f_y) - s(\underline{x});\ g_Z = r_Z(f_u) - s(\underline{x});\ g_A = r_A(f_{ub,Scher}) - s(\underline{x});\ g_L = r_L(f_{u,L}) - s(\underline{x}) \qquad (1)$$

OHL pylons are predominantly stressed by wind. Depending on the wind zone, a given characteristic stress $N_k = N(q_{0.98})$ includes a certain gust velocity pressure $q_k = q_{0.98}$ ($q_p(z)$ according to DIN EN 1991-1-4/NA [5]). There is a linear relationship between dynamic pressure and the acting force. This allows the wind speed associated with the stress to be determined.

$$\frac{N(q_{0.98})}{q_{0.98}} = \frac{s(q(v))}{q(v)} \rightarrow s(q(v)) = \frac{N(q_{0.98})}{q_{0.98}} \cdot q(v) \rightarrow \ s(v) = \frac{N(q_{0.98})}{\frac{1}{2} \cdot \rho_{Luft} \cdot v_{0.98}^2} \cdot \frac{1}{2} \cdot \rho_{Luft} \cdot v^2 = \frac{N(q_{0.98})}{v_{0.98}^2} \cdot v^2 \quad (2)$$

Now, taking into account the resistance side, the limit functions for the four failure possibilities considered can be formulated as follows.

$$g_D = \chi \cdot A_{eff} \cdot f_y - s(v) = \chi \cdot A_{eff} \cdot f_y - \frac{N(q_{0.98})}{v_{0.98}^2} \cdot v^2 \quad (3)$$

$$g_Z = A_{Ne} \cdot f_u - s(v) = A_{Ne} \cdot f_u - \frac{N(q_{0.98})}{v_{0.98}^2} \cdot v^2 \quad (4)$$

$$g_A = \sum A_{Scher} \cdot f_{ub,Scher} - s(v) = \sum A_{Scher} \cdot f_{ub,Scher} - \frac{N(q_{0.98})}{v_{0.98}^2} \cdot v^2 \quad (5)$$

$$g_L = \sum d \cdot t \cdot f_{u,L} - s(v) = \sum d \cdot t \cdot f_{u,L} - \frac{N(q_{0.98})}{v_{0.98}^2} \cdot v^2 \quad (6)$$

By defining the constructive properties, all points can be determined which are located on the limit state function g = 0. The limit state functions can then be represented graphically (see Figure 3).

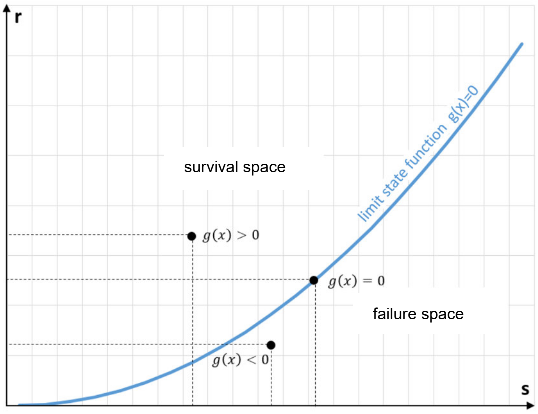

Figure 2: Limit state function in the original space

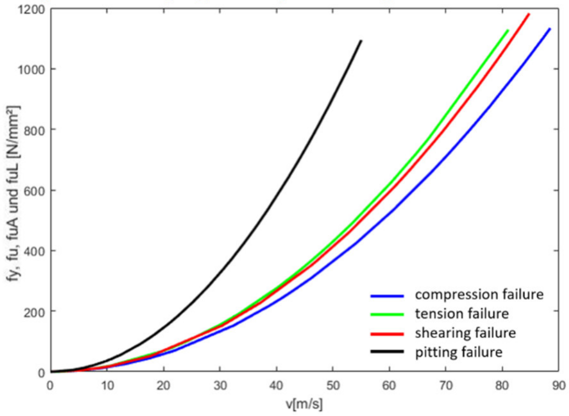

Figure 3: Example of limit state functions in the original space for four failure modes

2.5. Stochastic models

Stochastic models must be set up for probabilistic calculation. The term model indicates that there is a difference between actual frequencies of variable realisations and the probabilities predicted by mathematical formulae. However, suitable distributions, characterised by type and characteristic values, are described in the literature for the resistances to be applied in steel construction, e.g. the yield strength f_y, and the actions prevailing in OHL construction, e.g. the gust wind velocity v. The model is suitable if it is possible to determine the type and characteristic values by means of the mathematical formulae. The suitability is given if it could be shown by statistical fitting tests that there is no great difference between the actual frequency and the frequency predicted by the stochastic model of a variable realisation in the vicinity of the design point.

For the stochastic modelling of the gust wind speeds v, an extreme value distribution type III with variation coefficient $V = \sigma/m = 0.16$ and a positive curvature parameter τ = 0.06 is assumed according to VDE-AR-N 4210-4. This distribution is suitable for describing annual, largest impact realisations for which a maximum value x_{max} can be justified.

Cumulative distribution function [6]:

$$F_{(x)} = exp\left\{-\left[f_1 - f_2 \cdot \left(\frac{x - \mu_v}{\sigma_v}\right)\right]^{1/\tau}\right\} \quad (7)$$

Distribution density function:

$$f_{(x)} = \frac{f_2}{\sigma_v \cdot \tau} \cdot \left[f_1 - f_2 \cdot \left(\frac{x-\mu_v}{\sigma_v}\right)\right]^{\left(\frac{1}{\tau}-1\right)} \cdot exp\left\{-\left[f_1 - f_2 \cdot \left(\frac{x-\mu_v}{\sigma_v}\right)\right]^{1/\tau}\right\} \quad (8)$$

With: $f_1 = \Gamma(1+\tau) = 0.96874$ and $f_2 = \sqrt{\Gamma(1+2\tau) - {f_1}^2} = 0.07160$

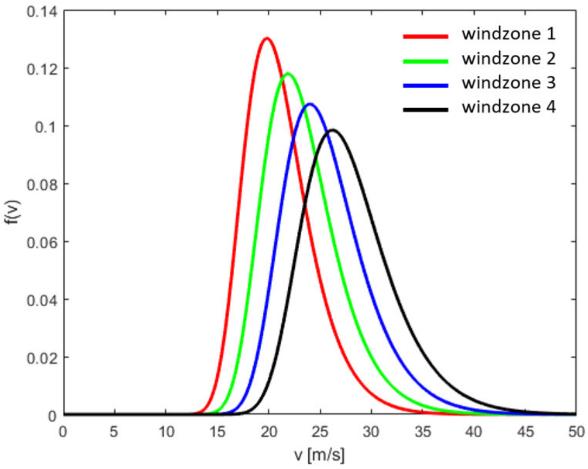

Figure 4: Distribution density functions Extreme value type III of the gust wind speeds (10m above ground level)

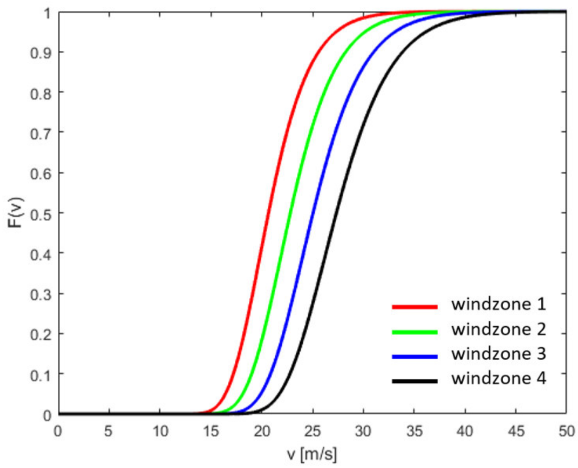

Figure 5: Cumulative distribution functions Extreme value type III of the gust wind speeds (10m above ground level)

The stressability of all failure modes is described by means of a lognormal distribution (VDE-AR-N 4210-4, 7.3.6). The distribution density function is given as follows:

$$f(x) = \frac{1}{\sqrt{2\cdot\pi}\cdot\sigma_{ln}\cdot x} \cdot e^{-\frac{(lnx-\mu_{ln})^2}{2\cdot\sigma_{ln}^2}} \quad (9)$$

The cumulative distribution function can be represented by the numerical calculation of the x-values. Taking into account the distribution parameters mean value m and standard deviation σ specified in VDE-AR-N 4210-4, the functions can be represented graphically:

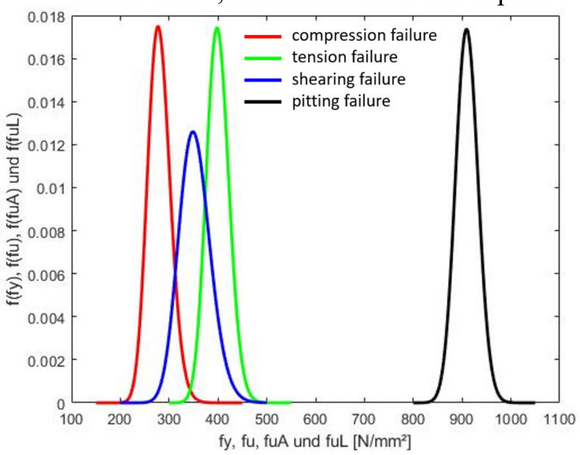

Figure 6: Distribution density functions of the lognormal distributed yield strength for S235

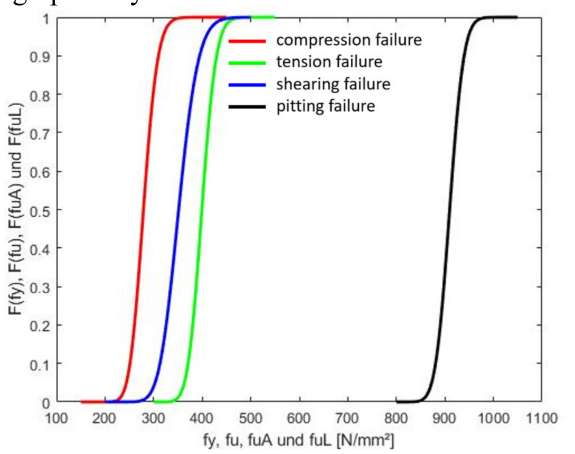

Figure 7: Cumulative distribution functions of the lognormal distributed strength

2.6. Standardisation

Limit states are then transformed into the standard space ($\Phi(u) = F(x)$) and the design point $\underline{u}_d = \underline{u}^*$ is determined, which is located on the limit state g(u) = 0 with the smallest distance to the origin. The length of this distance is the reliability index β of the component.

For the transformation, all function values in the original space F(x) are set equal to all function values in the standard space Φ(u). Thus, the function value of an x-variable in the original space is equal to the function value in the standard space. It can be assumed:

$$F(x) = \Phi(u) \rightarrow u = \Phi^{-1}[F(x)] \qquad (10)$$

After standardising the limit state for each failure mechanism, determine the shortest distance between g(x) = 0 and the origin. This distance corresponds to the reliability index β.

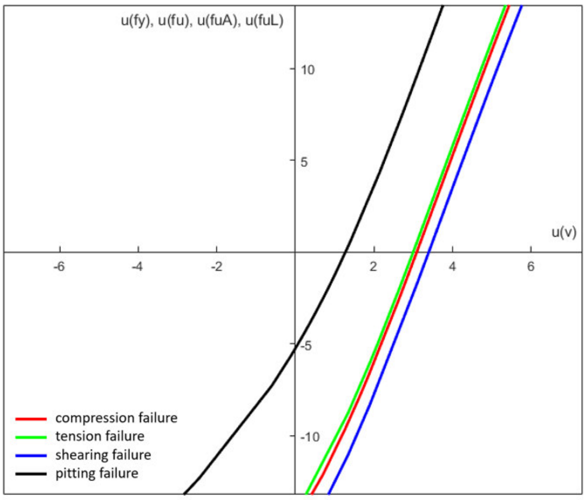

Figure 8: Example limit state functions in standard space for four failure modes

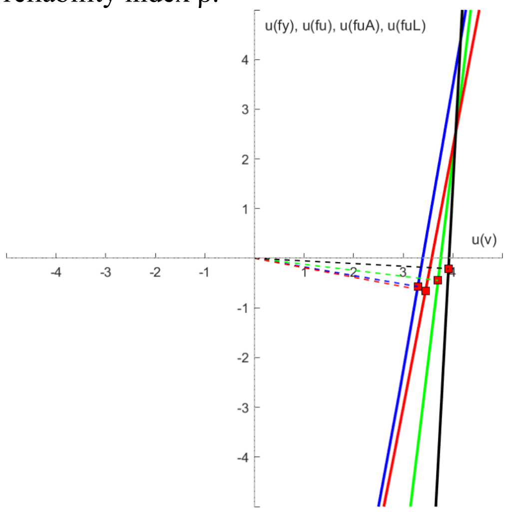

Figure 9: Reliability index

The relationship between the reliability index and the probability of failure is evident from the spatial representation of the standard space. With the help of the bivariate standard normal distribution Φ_2, which represents the dependence between two independent variables, the probability of failure can now be determined for each component.

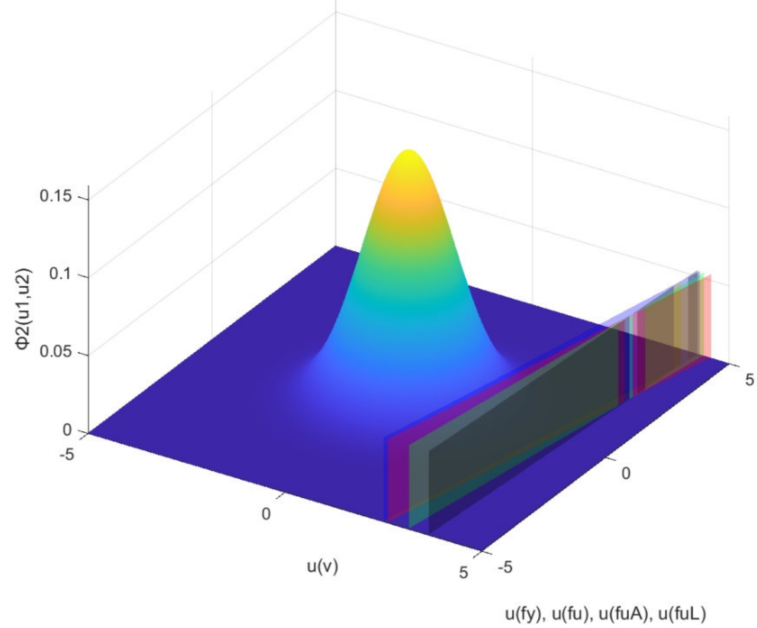

Figure 10: Bivariate standard normal distribution and limit functions in standard space

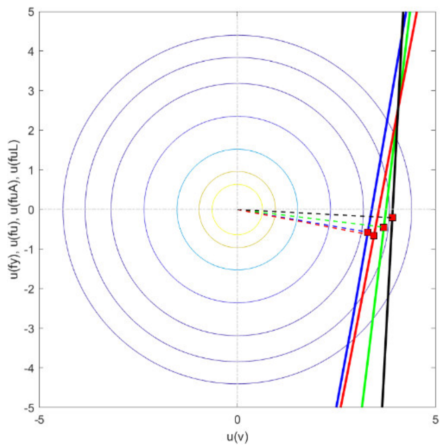

Figure 11: Bivariate standard normal distribution and limit functions in standard space (top view)

The limit state functions cut out a solid from a Gaussian bell at a distance from the standard normal distribution density perpendicular to the plane of the axis. Its size corresponds to the failure probability

P_f. The further g(x) = 0 is from the origin, the larger is β and the smaller is P_f. The point on g(x) = 0 at the shortest distance from the origin is called the design point. It is perpendicular to the origin.
The coordinates of the design point are calculated iteratively with the help of the Rackwitz-Fiessler algorithm [7]:

$$\underline{u}^{(k+1)} = \frac{\nabla g(\underline{u}^{(k)})}{\left\|\nabla g(\underline{u}^{(k)})\right\|^2} \cdot \left[(\underline{u}^{(k)})^T \cdot \nabla g\left(\underline{u}^{(k)}\right) - g(\underline{u}^{(k)})\right] \tag{11}$$

The partial derivatives contained are calculated by numerical differentiation. If no significant changes occur for all variables i between two iteration steps k and k+1, e.g. max | ui(k+1)-ui(k) | < 0.001, the iteration is terminated and the reliability index of a component j is calculated using the Pythagorean theorem:

$$\beta_j = \sqrt{\sum_i u_i^2} \tag{12}$$

The associated probability of failure is:

$$\beta = -\Phi^{-1}(P_f) \rightarrow P_f = \Phi(-\beta) \rightarrow P_f = \int_{-\infty}^{-\beta} \frac{1}{\sqrt{2\pi}} \cdot exp\left(-\frac{z^2}{2}\right) dz \tag{13}$$

The First Order Reliability Method (FORM) assumes a straight course of g(x) = 0. If this straight course actually exists, Figure 12 (shown on the left), the exact value of $P_f = \Phi(-\beta)$ is also known. However, g(x) = 0 is usually curved, so that the FORM solution is only an approximation. This approximation can be improved by calculating the curvatures of the limit state in x_d in addition to the coordinates x_d of the design point [8]. With knowledge of these curvatures, more exact SORM solutions can be calculated. In the case of a concave course, Figure 12 (shown in the centre), the value P_f calculated according to FORM is given by

$$P_f \approx \Phi(-\beta) \cdot \prod_{i=1}^{n-1} \left(1 - \frac{\varphi(\beta)}{\Phi(-\beta)} \cdot \kappa_i\right)^{-\frac{1}{2}} \tag{14}$$

is reduced, in the case of a convex course an increase takes place, Figure 12 (shown on the right).

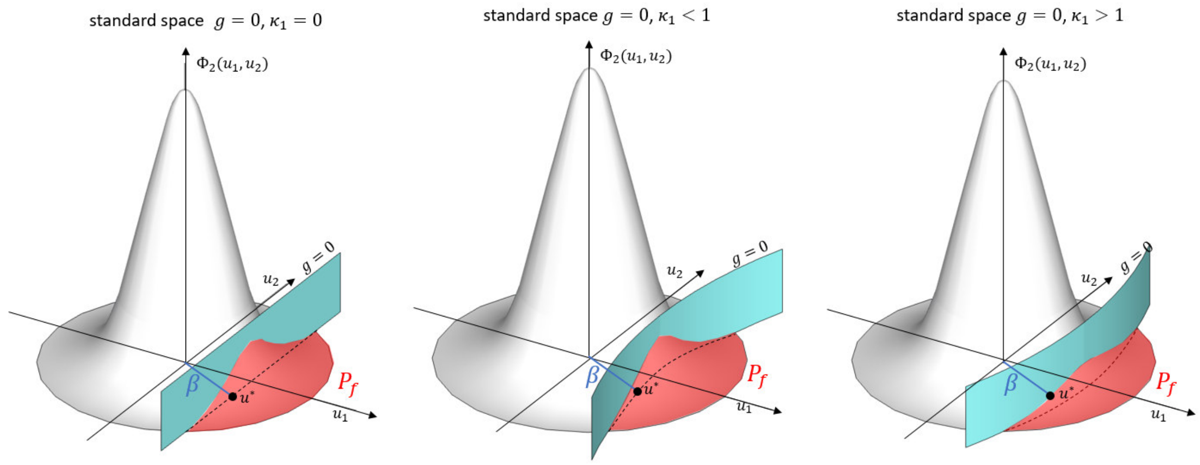

Figure 12: Representation of the limit state function in standard space

In addition, so-called sensitivities $\underline{\alpha} = \underline{u}_d / \beta$ are calculated from the design point and the reliability index, which represent the sensitivity of the system by a certain variable. The correlation matrix is determined from the sensitivity matrix:

$$\underline{\rho} = \underline{\alpha}^T \cdot \underline{\alpha} \tag{15}$$

2.7. Reliability index of a system

Once all component failure probabilities are known, the system failure probability must still be calculated. In general, it is not desirable for structures to have individual components that are defective, even if the entire structure has not yet collapsed. Otherwise, all defective components would have to be detected by inspection and replaced as part of repair measures to prevent overstressing of the remaining components. According to this pessimistic model of a series system, a system failure already exists when the first component fails. The system failure probability of a series system is always higher than the highest component failure probability, especially if the highest component failure probabilities can be found in many components and their safety margins are not strongly stochastically interdependent.
An elementary upper bound is the simple summation of all component failure probabilities. However, with a large number of components, an unrealistically high system failure probability is calculated, which could even exceed 1. This simple calculation does not take into account that for some variable realisations several components can fail at the same time. The probabilities for such variable realisations must then only be included once in the sum, not several times for all affected components. This probability is conform to the solid that $g_1 = 0$ and $g_2 = 0$ cut out from the Gaussian bell in Figure 13.

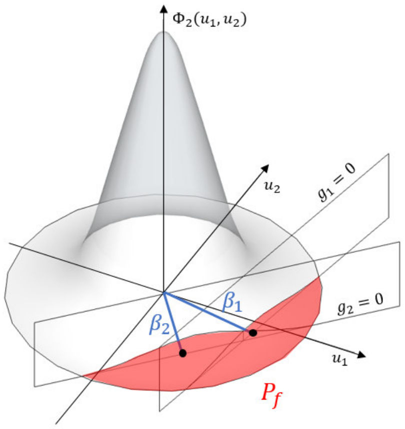

Figure 13: Failure space for two components

The effect of the failure of several components under the same variable realisation occurs particularly often when the safety margins of the components are strongly stochastically dependent on each other. This is the case with steel structures that are predominantly stressed by wind, because almost all components have a high sensitivity to variable wind speed. A very good way to capture this effect is the upper bound of the system failure probability according to Ditlevsen [9] and [10]. The probability of the simultaneous failure of a pair of components (j and b) is subtracted from the sum of the component failure probabilities using the standardised bivariate normal distribution:

$$P_{f,Sys} \le min\begin{cases} 1 \\ \sum_{j=1}^{k} P\left[g_j(\underline{x}) < 0\right] - \sum_{j=2}^{k} max_{(b<j)} \Phi_2(-\beta_j, -\beta_b; \rho_{j,b}) \end{cases} \tag{16}$$

The reliability index $\beta_{Sys} = -\Phi^{-1}(P_{f,Sys})$ is calculated from the failure probability of the overall system $P_{f,Sys}$, which is compared with the required reliability level of the site.
The safety present in the probabilistic calculation can be identified by back-calculating reliability coefficients. This is done using the reliability index and the characteristic value associated with it. A comparison with a semi-probabilistic calculation (Level I) is thus simplified.

3. Case Study

The theory of calculating reliability indices is presented below using an example. Consider two pressure diagonals in the wall perpendicular to the line of a supporting pylon as shown in Figure 14. Both bar forces are triggered exclusively by the gust velocity pressure. If the wind speed v should realise its 0.98 quantile $v_{0.98}$ = 32.6 m/s, diagonal D_1 reacts to this with 95 kN, diagonal D_2 with 80kN.

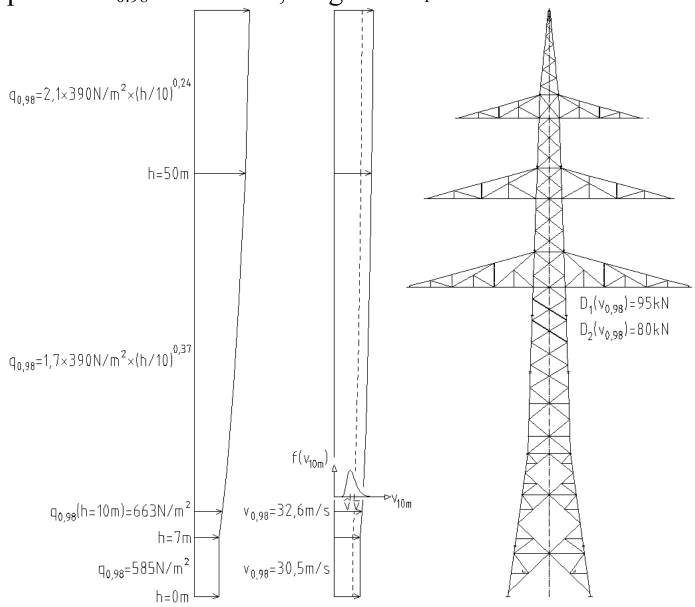

Figure 14: Two pressure diagonals in the wall perpendicular to a support pylon

For the diagonals considered as examples, the stochastic model is summarised in the table below:

Table III: Stochastic model of the capacities for pressure, tension, bolts and wind velocity 10 m above ground

Stochastic variable	**Distribution type**	**Mean value μ**	**Standard deviation σ**	**Maximum value x_{max}**
Effective area A_{eff1}	Constant	1550 mm²	-	-
Effectiv area A_{eff2}	Constant	1390 mm²	-	-
Reduction factor χ_1	Constant	0.410	-	-
Reduction factor χ_2	Constant	0.275	-	-
Yield strength f_{y1}	Lognormal	280 N/mm²	23 N/mm²	-
Yield strength f_{y2}	Lognormal	380 N/mm²	20 N/mm²	-
Wind velocity v	Extreme (III)	23.32 m/s	3.73 m/s	73.8 m/s

For the pressure diagonal D_1 according to Figure 14, for example, the iteration sequence is set up according to the Rackwitz-Fiessler algorithm in Table IV. The iteration starts for the wind speed v with the 0.98 quantile, (+2.054) = 0.98, for the yield strength with the 0.05 quantile, (-1.645) = 0.05.

Table IV: Design point search for the pressure failure mode of beam D_1

k	u_v	v [m/s]	$\frac{\partial g}{\partial u_v}$	u_{fy1}	f_{y1} [N/mm²]	$\frac{\partial g}{\partial u_{fy1}}$	g [N]	$\sqrt{\sum_i u_i^2}$	$\max\lvert u_i^{(k+1)}-u_i^{(k)}\rvert$
0	+2.054	32.57	-35296	-1.645	243.8	+12708	+59979	2.632	-
1	+3.385	41.44	-52518	-1.452	247.7	+12911	+3660	3.683	1.331
2	+3.592	42.903	-54673	-0.892	259.4	+13518	-4.07	3.701	0.560
3	+3.592	42.909	-54681	-0.888	259.5	+13522	+0.39	3.700	0.004
4	+3.592	42.909	-54681	-0.888	259.5	+13522	-0.45	3.700	0.000

After four iteration steps the iteration can be aborted. With the design point coordinates $u_{v_d} = +3.592$ ($v_d = 42.909 m/s$) and $u_{f_{y1_d}} = -0.888$ ($f_{y1_d} = 259.5 N/mm^2$) u_{fy1d}=-0.888 (v_d = 259.5 N/mm²) the most probable variable realisation triggering a failure is found. The design point is at g = 0. The reliability index is

$$\beta_{D_1} = \sqrt{3.592^2 + 0.888^2} = 3.700 \tag{17}$$

With this the probability of failure is $P_f = \Phi(-3.70) = 1.077 \cdot 10^{-4}$. The sensitivities of both variables are as follows

$$\begin{pmatrix} \alpha_v \\ \alpha_{fy1} \end{pmatrix} = \begin{pmatrix} u_v/\beta \\ u_{fy1}/\beta \end{pmatrix} = \begin{pmatrix} +3.592/3.700 \\ -0.888/3.700 \end{pmatrix} = \begin{pmatrix} +0.9708 \\ -0.2401 \end{pmatrix} \tag{18}$$

The reliability index of the pressure diagonal D_2 is

$$\beta_{D_2} = \sqrt{3.632^2 + 0.581^2} = 3.678 \tag{19}$$

The probability of failure $P_f = \Phi(-3.678) = 1.174 \cdot 10^{-4}$. The sensitivities of both variables are:

$$\begin{pmatrix} \alpha_v \\ \alpha_{fy2} \end{pmatrix} = \begin{pmatrix} u_v/\beta \\ u_{fy2}/\beta \end{pmatrix} = \begin{pmatrix} +3.632/3.678 \\ -0.581/3.678 \end{pmatrix} = \begin{pmatrix} +0.9875 \\ -0.1576 \end{pmatrix} \tag{20}$$

The exact SORM solution is now determined with the help of the correction factor. First, a new coordinate system is introduced whose first axis v_1 runs through the design point u_d. For the pressure diagonal D_1 the rotation matrix in the standard space coordinate system is

$$\underline{D} = \begin{bmatrix} +0.97076 & -0.24006 \\ -0.24006 & -0.97076 \end{bmatrix} \tag{21}$$

In this coordinate system, the matrix of the second and mixed derivatives of the limit state at the design point is calculated. Numerical differentiation for D_1 results in

$$\underline{B_u} = \begin{bmatrix} \frac{\partial^2 g}{\partial u_v^2} & \frac{\partial^2 g}{\partial u_v \cdot \partial u_{fy}} \\ \frac{\partial^2 g}{\partial u_v \cdot \partial u_{fy}} & \frac{\partial^2 g}{\partial u_{fy}^2} \end{bmatrix} = \begin{bmatrix} -9938 & 0 \\ 0 & 1109 \end{bmatrix} \tag{22}$$

and in the rotated v-coordinate system

$$\underline{B}_v = \underline{D}^T \cdot \underline{B}_u \cdot \underline{D} = \begin{bmatrix} -9301 & 2574 \\ 2574 & 472 \end{bmatrix} \tag{23}$$

The characteristic equation

$$det\left(\frac{\underline{\hat{B}}_v}{\frac{\partial g}{\partial v_1}} - \underline{\kappa} \cdot \underline{E}\right) = 0 \tag{24}$$

is solved for the vector $\underline{\kappa}$ of curvatures of the limit state at the design point. $\underline{\hat{B}}_v$ is the matrix according to $\underline{B}_v$ after deleting the first row and the first column, i.e. the number of curvatures is smaller than the number of variables by 1. In the case of the pressure diagonal D_1, $\underline{\hat{B}}_v$ degenerates to the number 472. The first derivatives of the limit state at the design point are

$$\nabla g(\underline{v}) = \underline{D} \cdot \nabla g(\underline{u}) = \begin{bmatrix} +0.97076 & -0.24006 \\ -0.24006 & -0.97076 \end{bmatrix} \cdot \begin{bmatrix} -54681 \\ 13522 \end{bmatrix} = \begin{bmatrix} -56328 \\ 0 \end{bmatrix} \tag{25}$$

Thus, the characteristic equation results in

$$det\left(\frac{\underline{\hat{B}}_v}{\frac{\partial g}{\partial v_1}} - \underline{\kappa} \cdot \underline{E}\right) = \frac{472}{-56328} - \kappa \cdot 1 = 0 \rightarrow \kappa = -0.00838 \tag{26}$$

The SORM improved failure probability for pressure diagonal D_1 is then

$$P_f \approx \Phi(-3.700) \cdot \left(1 - \frac{4.248 \cdot 10^{-4}}{1.077 \cdot 10^{-4}} \cdot (-0.00838)\right)^{-\frac{1}{2}} = 1.077 \cdot 10^{-4} \cdot 0.9839 \\ = 1.059 \cdot 10^{-4} \tag{27}$$

There is a slightly concave course of the limit state at the design point. The probability of failure is actually somewhat smaller than when assuming a straight-line limit state. The reliability index β increases minimally to

$$\beta = -\Phi^{-1}(1.059 \cdot 10^{-4}) = 3.705 \tag{28}$$

The correlation matrix is obtained by multiplying the transposed sensitivity matrix $\underline{\alpha}^T$ by the sensitivity matrix. For variables on which a reliability index does not depend, use 0.

$$\underline{\varrho} = \begin{bmatrix} 0.9708 & -0.2401 & 0 \\ 0.9875 & 0 & -0.1576 \end{bmatrix} \cdot \begin{bmatrix} 0.9708 & 0.9875 \\ -0.2401 & 0 \\ 0 & -0.1576 \end{bmatrix} = \begin{bmatrix} 1 & 0,9586 \\ 0,9586 & 1 \end{bmatrix} \tag{29}$$

With $\Phi_2(-\beta_j, -\beta_b;\ \varrho_{j,b}) = \Phi_2(-3.700, -3.678;\ 0.9586) = 6.488 \cdot 10^{-5}$ as the probability for the occurrence of variable realisations that cause both components to fail, the upper bound of the system failure probability is given by

$$P_{f,Sys} \leq 1.059 \cdot 10^{-4} + 1.166 \cdot 10^{-4} - 6.488 \cdot 10^{-5} = 1.576 \cdot 10^{-4} \tag{30}$$

The system failure probability according to Ditlevsen is about 1.5 times higher than the highest component failure probability. A simple summation of the component failure probabilities would lead to a potential doubling. The reliability index of the system decreases compared to the smallest component reliability index from 3.678 to

$$\beta = -\Phi^{-1}(1.576 \cdot 10^{-4}) = 3.602 \quad (31)$$

4. Conclusion

Efficient and sustainable use of resources is becoming increasingly important in many areas of society and the environment, and is therefore also a goal of grid operation.
With the probabilistic verification method, it is possible to maintain the existing grid according to individual safety levels with optimised labour. With the stochastic models of probabilistic, actions and resistances can be described very accurately, whereby an efficient stability verification of lattice pylons can be carried out.
The development of the probabilistic safety concept for lattice pylons can be applied to other areas of the energy grid. It thus offers great technical potential to ensure electrical supply security in the long term with a stable power grid.

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